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DETERMINATION OF ALL THE GROUPS OF ORDER 168.

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The first two composite numbers which are orders of simple groups are 60 and 168. Burnside determined the possible groups of the former of these orders in his Theory of Groups. The present paper is devoted to those of the latter order.

According to Sylow's theorem a group of order 168 contains either one or eight subgroups of order 7. If it contains 8 subgroups of order 7 it must permute them according to a transitive group of degree 8. As this transitive group involves substitutions of order 7 it must be doubly transitive. Hence such a group of order 168 must be isomorphic with a primitive group of degree 8 whose order is either 168 or 56. In the former case there is a simple isomorphism and the two possible groups are well known.* We proceed to prove that there is only one group of order 168 that contains 8 subgroups of order 7 and is isomorphic with the primitive group of degree 8 and order 56.

Since this primitive group contains an invariant subgroup of order 8 which involves 7 operators of order 2, such a group (G) contains an invariant subgroup (H) of order 24 which includes only operators of orders 2, 3, and 6. It contains only one subgroup of order 3. The two operators of order 3 must be invariant since the primitive group of degree 8 and order 56 does not include a subgroup of order 28. Hence H is composed of 7 operators of order 2 and 14 of order 6,

*Hoelder, *Mathematische Annalen*, Vol. 40, 1892, page 75; Cole, *Bulletin of the New York Mathematical Society*, Vol. 2, 1893, page 189.